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# Statistical measure of non-classicality as it pertains to noise

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## Abstract:

The search for evidence of the boundary between classical and quantum mechanics is a quest of fundamental and enduring importance. Using semiclassical arguments, we demonstrate that the boundary between the classical and quantum domains can be defined in terms of the specific characteristics of a delimiter parameter related to the notions of I noise ii) Hisami distribution functions, iii) Where’s entropy, and iv) escort distributions.

## Introduction

The search for evidence of the boundary between classical and quantum mechanics is a quest of fundamental and enduring importance. Like phase-space points, coherent states may be thought of as discrete entities. The topic of how far the features of an arbitrary quantum state defined by the density operator deviate from those of coherent states is a natural one to consider. In this regard, Kenfack and Sikorski [2] investigate the negative of the Wigner function and question whether there is any parameter that may legitimately express the degree of non-classicality of. With this note, we want to provide a new perspective within the field of semiclassical statistics [3], specifically with respect to the methods and data from quantum optics. You may reach us at plastino@fisica.unlp.edu.ar. medical procedures involving the ear. Our thoughts revolve on "noise," which has been shown to be a very revealing factor in particle-wave duality [4]. Variations in electromagnetic fields are distinct depending on whether the energy is transmitted as waves or particles. Classical waves' energy fluctuations grow proportionally to their mean energy, but classical particles' energy fluctuations grow at a rate equal to the square root of their mean energy. To accommodate the fact that a photon is neither a classical wave nor a classical particle, both linear and square-foot contributions are required. At optical frequencies, the square-root (particle) contribution is more significant than the linear (wave) contribution [5-7]. In the 1960s, it was shown that fluctuations may tell the difference between the radiation from a laser and that from a black body, hence expanding the diagnostic potential of photon-noise. The wave contribution to the fluctuations is zero for the former and negligible for the latter (a dark body). Quantum optics has advanced to the point that noise measurements are routine, and Glauber's quantum theory of photon statistics is required reading. For this reason, coherent states [5, 6] take on a pivotal role in quantum optics [1, 8, 9], since they are the states of a harmonic oscillator system that most closely resemble the classical motion of a particle in a quadratic potential. In fact, the techniques of quantum optics derive much of their power from their ability to exploit classical analogues and, in particular, comparisons with classical noise theory, which allow reduction of purely harmonic systems to non-operator ones, via phase space methods [1, 8], in which the essentially quantal nature of the problem is transcribed in terms of the interpretation of apparently classical variables, with coherent states [9] playing the lead role. Again, that function will be called upon within the bounds of semiclassical methods to address the question given in the opening paragraph. In this talk, I'll demonstrate how the peculiarities of a semiclassical delimiter parameter linked to the ideas of

## Preliminaries

## Wheel entropy and Hisami distributions

As an example of a semiclassical idea, we look to Wheels' entropy W, which provides a practical metric for gauging the degree to which phases are locally coherent [1, 10, 11]. The relevant definition is as follows:

$$W = -\int \frac{dx\,dp}{2\pi\hbar}\,\mu(x,p)\,\ln\mu(x,p), \qquad (1)$$

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where (pop) = z||z is the density matrix of the system, and (pop) is a "semi-classical" phase-space distribution function [1, 9]. By satisfying the equation adze = zizz, coherent states are eigenstates of the annihilation operator a. Distribution (pop) is scaled in the way

$$\int (\mathrm{d}x\,\mathrm{d}p/2\pi\hbar)\,\mu(x,p) = 1, \qquad (2)$$

the Hisami distribution [12] is a common name for this kind of distribution. Wheel entropy is the same as the "classical entropy" (1) of a Wigner distribution, as shown by the final two equations. Actually, (pop) is a Wigner-distribution DW spread out across a bounded area of phase space of size. This was shown in [10]. Smearing makes (pop) a positive function even though DW does not have a positive character. Hisami's semi-classical probability distribution is a subset of the probability space [10] for the simultaneous but approximate localization of position and momentum in phase space. Because of this inequality, we can see that the uncertainty principle is at work.

$$1 \leq W, \qquad (3)$$

in which Wheel made a guess [11] and Lieb proved it [13]. The Gibbs canonical distribution and its related "thermal" density matrix are used for the typical presentation of equilibrium in statistical mechanics, and are given by.

$$\rho = Z^{-1}\mathrm{e}^{-\beta H}, \qquad (4)$$

using the partition function Z = Tr(Eh), the inverse temperature T = = 1/Kbit, and kB as the Boltzmann constant. Take any Hamiltonian H with eigen-energies End and eigenstates |n, and you can easily put out a formula for W. (n stands for a collection of all the pertinent quantum numbers required to label the states). Writing is always an option.

$$\mu(x,p) = \frac{1}{Z}\sum_{n} \mathrm{e}^{-\beta E_n}|\langle z|n\rangle|^2, \qquad (5)$$

By combining Eq. (5) with Eq. (6), we have a practical method for arriving to W. (1). Coherent states have the form [1] in the case of the harmonic oscillator.

$$|z\rangle = \mathrm{e}^{-|z|^2/2}\sum_{n=0}^{\infty}\frac{z^n}{\sqrt{n!}}|n\rangle, \qquad (6)$$

where End = (n + 1/2), where n = 0; 1; where |n are an orthonormal collection of eigenstates. In this case, the helpful analytic expressions found in Ref. [10] may be used.

$$\mu(z) = (1 - \mathrm{e}^{-\beta\hbar\omega})\,\mathrm{e}^{-(1-\mathrm{e}^{-\beta\hbar\omega})|z|^2}, \qquad (7)$$

$$W = 1 - \ln(1 - \mathrm{e}^{-\beta\hbar\omega}). \qquad (8)$$

The entropy, at its lowest point at T 0, expresses just quantum fluctuations with the value W = 1. In contrast, when T is below the critical temperature, the entropy approaches the value\

**An indicator of noise: the Mandel parameter**

The so-called Mandel parameter, introduced by [7], is a handy noise-indicator of a non-classical field.

$$Q = \frac{(\Delta N)^2}{\langle \hat{N}\rangle} - 1 \equiv F - 1, \qquad (9)$$

This is connected to the normalised variance of the photon distribution, F = (N2)2/(N2)N, commonly known as the quantum Fano factor F [14]. As the photo-count noise of the produced light is less for F 1 (Q 0) than it is for coherent (ideal laser) light of the same intensity (F = 1; Q = 0), the light is referred to as sub-Poisoning, while for F > 1 (Q > 0) it is referred to as super-Poisoning. It's obvious that reducing the Fano effect is a goal. The Mandel parameter disappears (Q = 0 and F = 1) in a coherent state, also known as a pure quantum state. When the Heisenberg uncertainty relation is maximised and the uncertainty is the same in all four quadrature components, a coherent field is the quantum-state most similar to a classical field. Here we ask: how near can we go to Q = 0 (or F = 1) using semiclassical methods? Finding where classical and quantum fields meet with this solution should be rather straightforward. If that's the case, then it's evident that both Q and F serve as non-classicality markers. Indeed, in a thermal state, Q > 0 and F > 1, indicating a more dispersed photon distribution than the Poisoning model predicts. At the limit of Q = 0, (F = 1), the photon distribution becomes smaller than a Poisson-PDF, and the corresponding state is not classical. Number states are the simplest kind of non-classical states. This is because they are eigenstates of the photon number

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operator N, therefore fluctuations in N disappear and Q = 1 (F = 0) appears in the Mandel parameter [15]. Here, we'll make a semiclassical connection between these thoughts.

## Present considerations: semiclassical Q−evaluation

The HO-Mandel parameter may be rewritten as follows, taking into consideration the connection between the number operator and the Hamiltonian of the harmonic oscillator H through N = H/1/2.

$$Q = F - 1 = \frac{(\Delta\hat{H})^2}{\hbar\omega\langle\hat{H}\rangle - \hbar^2\omega^2/2} - 1, \qquad (10)$$

where we have relied on the formula H = |z|2 [3]. From here on out, Sc, a semiclassical variant of Mandel's paramo

$$F^{sc} - 1 = Q^{sc} = \frac{(\Delta_\mu N)^2}{\langle\hat{N}\rangle_\mu} - 1, \qquad (11)$$

where ⟨:::⟩ The subindex indicates that the Hisami distribution (7) has been used as the weight function, and the symbol stands for the semiclassical mean value of any generic observable. It is therefore straightforward to observe that Screads.

$$Q^{sc} + 1 = F^{sc} = \frac{2}{(1 - e^{-\beta\hbar\omega})(2 - (1 - e^{-\beta\hbar\omega}))} \geq 2, \quad (12)$$

Note that the semiclassical technique prevents us from obtaining the Q = 0 value (for an explanation, see the note below Eq. Now we turn to expound on some other resources that, it is hoped, will throw even more light on this critical (as will be shown) topic.

## Escort-Fano factor:

There is an infinite family of related probability distributions (PDs) ft(x) provided by for each PD f(x).

$$f_q(x) = \frac{f^q(x)}{\int f^q(x)\,dx}; \qquad (q \in \mathcal{R}), \qquad (13)$$

These have been shown to be very helpful in the study of nonlinear dynamical systems because they are typically better at revealing aspects of the system than the original distribution [16]. Using the idea of escort distribution (ED) with semiclassical Hisami distributions may help ameliorate the situation stated above. As a result, it is possible to use quantitative Hisami distributions (Quds) q(pop) to glean "better" semiclassical information.

$$\gamma_q(x,p) = \mu(x,p)^q \Big/ \left( \int \frac{d^2z}{\pi}\, \mu(x,p)^q \right), \qquad (14)$$

using the formula d2z/ = dad/2 whose HO-analytic form is available from Ref. [17], i.e.,

$$\gamma_q(z) = q(1 - e^{-\beta\hbar\omega}) \exp\left[-q(1 - e^{-\beta\hbar\omega})|z|^2\right]. \qquad (15)$$

For the appropriate Hamiltonian-moments, we now calculate the expectation values in Eq. (10), using q as the weight function.

$$\langle H\rangle_{\gamma_q} = \int \frac{d^2z}{\pi}\gamma_q(z)\hbar\omega|z|^2 = \frac{\hbar\omega}{q(1 - e^{-\beta\hbar\omega})}, \qquad (16)$$

$$\langle H^2\rangle_{\gamma_q} = \int \frac{d^2z}{\pi}\gamma_q(z)\hbar^2\omega^2|z|^4 = \frac{2\hbar^2\omega^2}{q^2(1 - e^{-\beta\hbar\omega})^2}, \qquad (17)$$

$$(\Delta H)^2_{\gamma_q} = \frac{\hbar^2\omega^2}{q^2\left[1 - \exp(-\beta\hbar\omega)\right]^2}, \qquad (18)$$

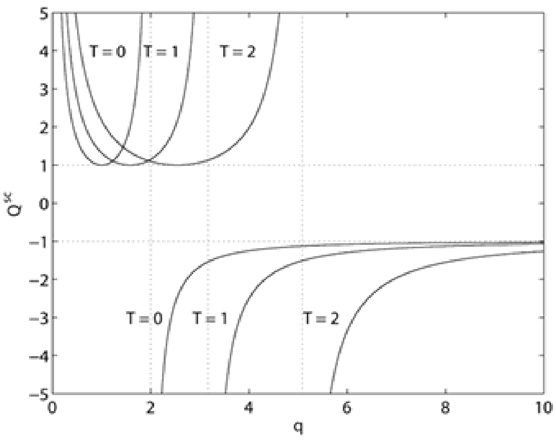


**Figure 1. Mandel parameter Sc evaluated semi classically by recourse to escort distributions of order q at different temperatures T (given in ω−units).**

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that the Mandel parameter (Fano factor) may be expressed as a "escort"-type expression:

$$Q_q^{sc} + 1 = F_q^{sc} = \frac{2}{q\,(1 - e^{-\beta\hbar\omega})\,(2 - q(1 - e^{-\beta\hbar\omega}))}. \quad (19)$$

The value of Sc is less than one as q approaches unity. Fig. 1 illustrates how the acquisition of q provides the extra degree of freedom necessary to achieve the required negative values of the Mandel parameter. More factors are required for interpreting these findings. Let's start with the sportwear entropy constructed from the distributions q, which looks like this: (found in [18])

$$W_q = W - \ln q, \quad (20)$$

as a result, values cannot be negative. In order to ensure that the information measure We satisfies Lieb's constraint and is positive (i.e., 1 We 0), as implied by Eq. (20) and the HO-analytic formula (8), we must limit the escort-degree orange to 1 q e 2:7182818. Even yet, the aforementioned space may be further constrained by taking into account other, more subtle factors. We now use the idea of the participation ratio R of a density operator (which provides the number of pure states that enter [19, 20]) to get this conclusion:

$$\mathcal{R} = 1/Tr(\rho^2); \qquad [1 \leq \mathcal{R} \leq \infty]. \quad (21)$$

Our next step is to devise a semiclassical "equivalent-notion" by completing a similar computation using the escortHisami distribution q of the harmonic oscillator. The resulting harmonic oscillator distribution would be escortHisami q. As a result of this,

$$\mathcal{R}_q^{HO} = \frac{1}{\int \frac{d^2z}{\pi}\, \gamma_q(z)^2} = \frac{2}{q(1 - e^{-\beta\hbar\omega})}. \quad (22)$$

For example, consider the expression RHO 2 q (1 e): (22) q=1(T = 0) = 2. Even in the "best case" situation, our density operator (4) has at least two pure states, which prevents us from semi classically obtaining Q =0 in El (12). Thus, at absolute zero, q = 2 is a direct consequence of invoking R 1: When temperature increases, the region of allowable purview moves "to the right," and eventually passes the value of 2. For T = 0, a more restricted area F of allowable values for q follows, namely, F = [1 q 2], which is relevant to our current topic as a look at Fig. 1 will demonstrate. The rightward expansion of F is a direct result of the growth of T, as previously mentioned. In Fig. 1, we see that the idea of escort distributions of order 2 q e allows us to reach the world of negative (and consequently quantum) values of the Mandel parameter Q in a semiclassical fashion. Yet, the quantum-classical boundary starts at Sc = 1, making the physical (quantum) region 1 Q 1 inaccessible to our modified semiclassical method. Negative fluctuations, as implied by the Sc 1 values in Fig. 1 [Cf. Eq. (9)], are not physically plausible and so cannot be taken seriously. Keep in mind that Sc = 1 is obtained for q = (for all temperatures T), but this is also not physically plausible since the corresponding escort-Hisami distribution would be a delta in phase space, which goes against the uncertainty principle. For the sake of ensuring that our findings are not only a Hisami artefact, but we will also now attempt to solve the identical problem using a different strategy.

## Escort thermal Wigner distribution

It is well-known that for any generic density matrix, the Wigner distribution may be written as [22].

$$f_W(x,p) = \int_{-\infty}^{\infty} ds\, e^{ips/\hbar} \left\langle x - \frac{s}{2} \middle| \rho \middle| x + \frac{s}{2} \right\rangle, \quad (23)$$

which is scaled such that R(d2z/) few(pop) = 1. Thus, the appropriate analytic equation for the thermal density of a harmonic oscillator is [22].

$$f_W(x,p) = 2\tanh(\beta\hbar\omega/2)\, e^{-2\tanh(\frac{\beta\hbar\omega}{2})|z|^2}, \quad (24)$$

using the formula |z| = x242 x + p242 p, where x = /2m and p = /2m. It is at this time that we provide a formal definition of the escort-thermal Wigner

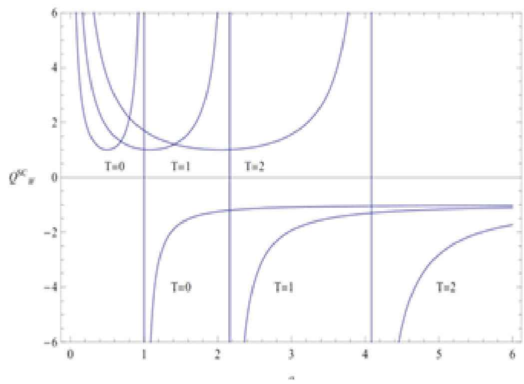

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***Figure 2. Mandel parameter Sc W evaluated semi classically by recourse to escort distributions of order q at different temperatures T (given in ω−units).***

## distribution as follows:

$$\zeta_q(x,p) = f_W(x,p)^q / \left( \int \frac{d^2z}{\pi} f_W(x,p)^q \right), \qquad (25)$$

Thus, if we do an integration across phase space, we will

$$\zeta_q(x,p) = 2q \tanh(\beta\hbar\omega/2) e^{-2q\tanh(\beta\hbar\omega/2)|z|^2}. \qquad (26)$$

After this evaluation, the Mandel parameter looks like

$$Q_W^{SC} = \frac{1}{2q\tanh(\beta\hbar\omega/2)\left(1 - q\tanh(\beta\hbar\omega/2)\right)} - 1. \qquad (27)$$

QSC W against q is shown in Fig. 2. Figure 2 shows that the behaviour of Figure 1 is recovered, although with a modified q-dependence.

## Conclusions

Where is the value in our oversimplification of the escort industry? So, to be able to determine that, when the escort degree q adopts certain values, pretty unusual things happen that plainly exhibit no classicality (our goal in this communication). Since Sc = 1 marks the boundary between the classical and quantum worlds, such eccentricity appears to indicate that we have crossed over into the latter. First, observe what occurs at q = 2; T = 0, the initial condition under which negativity is tenable. It is important to realise that the resulting semiclassical escort-Hisami distribution for e q 2 cannot be associated à la (22) to a quantal distribution function derived from a density operator because in that case the participation ratio would be less than unity, a limit value only reached by pure states. Nonetheless, this does highlight a contradiction between the quantum regime and escort distributions of degree > 2, which has of little importance for the semiclassical method because it is not a quantum one. When q = 2, Sc suddenly leaps from positive to negative infinity (recall that T = 0), thus the transition is anything from smooth. By substituting q = 2/[1 exp()] for q = 2, these arguments remain true even at limited temperatures. Second, the escort distributions transform into a Dirac's delta in phase space [21], where we get the quantal Qis = 1. The same qualitative pattern is shown when the thermal-Wigner distribution is used in lieu of the escort-Hisami distribution, despite the fact that the related q-values deviate somewhat from the Hussaini ones. Consequently, we run into the aforementioned peculiar behaviours if we want for our semi classically assessed noise estimator Q to accept values associated with the quantal domain. In this regard, a hypothesis may be formulated. Abnormalities in the behaviour of semiclassical quantities might point to non-classicality. Although we conclude that it is impossible to reach the quantum regime through a semiclassical approach, we do conclude that our method "senses" the presence of such a regime. In addition, non-classicality can be "visualised" in classical terms, despite the apparent contradiction: non-classicality necessitates zero-fluctuations in the particle-number alongside finite ones in phase-space location, which is impossible in classical physics (owing to the Dirac's delta at q = ). Therefore, we conclude that the degree q of the escort distribution of a Hisami distribution is a semiclassical signal of non-classicality, which we have discovered in our study. With q equal to 2, we move out of the classical area. In this case (with q between 0 and 2), the participation ratio is less than 1, hence it is impossible to correlate a quantum probability distribution (such the ||2sort) with the semiclassical PDs that describe the situation. For q =, we seem to obtain the truly quantum case of zero particle-number fluctuations, at the expense of breaking the uncertainty principle. So, it has been concluded that the escort-semiclassical method displays certain characteristics that facilitate "visualising" the classical-quantum boundary.

## References

*[1] R. J. Glauber, Phys. Rev. 131, 2766 (1963)*

*[2] A. Kenfack, K. Sikorski, J. Opt. B-Quantum S. O. 6, 396 (2004)*

International Journal of Management Research and Business Strategy

ISSN 2319-345X

www.ijmrbs.com

Vol. 13, Issuse. 1, Jan 2024

*[3] F. Pennini, A. Plastino, Phys. Rev. E 69, 057101 (2004)*

*[4] C. Beehacker, C. Schonberger, Phys. Today 56, 37 (2003)*

*[5] D. F. Walls, G. J. Milburn, Quantum optics (Springer, New York, 1994)*

*[6] P. Mestre, M. Sargent III, Elements of quantum optics (Springer, New York, 1990)*

*[7] L. Mandel, E. Wolf, Optical coherence, and quantum optics (Cambridge University Press, New York, 1995)*

*[8] E. C. G. Sudarshan, Phys. Rev. Lett. 10, 277 (1963)*

*[9] J. R. Klauder, B. S. Sagest, Coherent states (World Scientific, Singapore, 1985)*

*[10] A. Anderson, J. J. Halliwell, Phys. Rev. D 48, 2753 (1993)*

*[11] A. Wheel, Rep. Math. Phys. 16, 353 (1979)*

*[12] K. Hisami, Proceedings of the Physics and Mathematics Society of Japan 22, 264 (1940)*